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Optimal Markov Model Relating *Two* Time-Lagged Outcomes

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This paper demonstrates the use of maximum-accuracy weighted Markov analysis to model the relationship between two time-lagged variables—serial ratings of pain during the day and subsequent quality of sleep at night—for an individual.

If used to model an ordinal outcome for a single outcome variable, a Markov Model may be specified using the lowest granularity ordered configuration.[1] For example, to model a serial symptom rating made by a single person, on each sequential trial the rating is coded as being lower (less severe symptom rating at time *i*+1 *vs*. time *i*), or unchanged/up (unchanged or more severe symptom rating at time *i*+1 *vs*. time *i*). As seen in Figure 1, "Down" (D) indicates the symptom rating was less severe, and "Not Down" (ND) indicates the symptom rating was either unchanged or more severe.

Figure 1: Least-Granular Ordered Markov Configuration for *One* Symptom Rating

Symptom X Time *i*	**Symptom X Time *i*+1:** Down	Not Down
Down	D-D	D-ND
Not Down	ND-D	ND-ND

Two possible ordered model configurations are *weighted* and *unweighted*. Unweighted designs contrast relative change in symptom rating over successive ratings. Weighted designs weight each entry in the Markov table by the absolute value of the change in symptom rating: entries with a zero weight are thus omitted from weighted analysis.[2]

This article proposes the use of weighted Markov analysis to model the association of *two* serial ordinal ratings made by a single person, as indicated in Figure 2.

Figure 2: Least-Granular Ordered Markov Configuration for *Two* Symptom Ratings

Symptom X Time *i*	**Symptom Y Time *i*+1:** Down	Not Down
Down	D-D	D-ND
Not Down	ND-D	ND-ND

Data

Data for this study were obtained vis-à-vis the Self-Monitoring And Review Tool (SMART), an interactive internet-based self-monitoring and feedback system which helps individual users to discover and monitor links between their own health-related behaviors, management strategies, and symptom levels over time.[3,4] SMART

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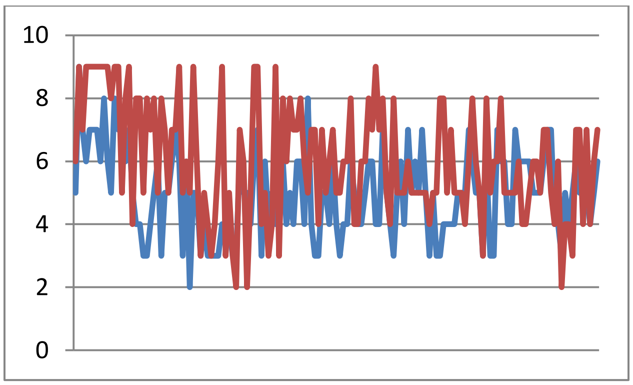
involves longitudinal collection and optimal statistical analysis of an individual's self-monitoring data, and delivery of personalized feedback derived from the data. A total of 148 daily pain (during the day) and sleep status (during the night) ratings made by a single individual using a 10-point Likert-type scale (1=not at all bothersome; 10=extremely bothersome) were used in analyses reported herein. A plot of the symptoms over time is presented in Figure 3.

Figure 3: Patient's Raw Pain (Blue) and Sleep Difficulty (Red) Ratings by Time



Unweighted Analysis: Raw Data

The pain (rating at time *i*) and sleep (rating at time *i*+1) series were used to construct a unit-weighted Markov transition table (Table 1; in ODA, ordinarily rows indicate the class variable but in Markov analysis the columns indicate the class variable). Data were assessed using non-directional ODA testing the exploratory hypothesis that data fall into one or the other diagonal. The ODA model indicated stationarity (a lower symptom rating at t_i predicts a lower symptom rating at t_{i+1}, and *vice versa*). The model yielded 49.09% sensitivity when classifying Down days, and 69.57% when classifying Not Down days: this result was statistically significant ($p<0.034$), indicating relatively weak predictive accuracy (ESS=18.66; 0=chance; 100=perfect accuracy). The model and the results were both stable in leave-one-out (LOO) one-sample jackknife analysis ($p<0.019$).[5]

Table 1: Transition Table for Unweighted Change in Sleep Rating as a Function of Change in Pain Rating

Symptom X Time *i*	**Symptom Y Time *i*+1** Down	Not Down
Down	27	28
Not Down	28	64

Weighted Analyses: Raw Data

Three weighted analyses were conducted. First, every event in the transition table was weighted by the absolute value of the difference in pain rating between the index (t_i+1) *vs*. prior (t_i) day: weights must be positive, so the sample was reduced by the number of events with a difference score of 0. The resulting ODA model indicated stationarity, yielding the transition table seen in Table 2. For the weighted model the weighted sensitivity for Not Down (87.78) and for Down (56.18) exceeded unit-weighted findings: the resulting weighted ESS=43.96 is a moderate effect—statistically significant ($p<0.0001$) and stable in LOO analysis ($p<0.00004$).

Table 2: Transition Table for Change in Sleep Rating as a Function of Weighted Change in Pain Rating: Raw Data

Symptom X Time *i*	**Symptom Y Time *i*+1** Down	Not Down
Down	27	6
Not Down	28	44

Second, every event in the transition table was instead weighted by the absolute value of the difference in sleep rating between the index (t_i+1) *vs*. prior (t_i) day. The resulting ODA model indicated stationarity (Table 3). For the weighted model the weighted sensitivity for Down (62.63) but not for Not Down (57.42) exceeded the unit-weighted findings: resulting weighted ESS=20.05 is a relatively weak effect, which is not statistically significant ($p<0.083$).

Table 3: Transition Table for Change in Sleep Rating as a Function of Weighted Change in Sleep Rating: Raw Data

Symptom X	**Symptom Y**	
	Time *i*+1	
Time *i*	Down	Not Down
Down	27	28
Not Down	15	42

Finally, every event in the transition table was instead weighted by a complex weight defined as the product of the absolute value of the difference in sleep rating between the index (t_i+1) *vs*. prior (t_i) day and the absolute value of the difference in pain rating between the index (t_i+1) *vs*. prior (t_i) day. The resulting ODA model indicated stationarity (Table 4). For the weighted model the weighted sensitivity for Down (70.35) and Not Down (88.67) exceeded unit-weighted findings: the resulting weighted ESS=59.02 is a relatively strong effect, which is statistically significant ($p<0.0001$) and stable in LOO analysis ($p<0.00002$).

Table 4: Transition Table for Change in Sleep Rating as a Function of Weighted (Change in Sleep x Change in Pain) Rating: Raw Data

Symptom X	**Symptom Y**	
	Time *i*+1	
Time *i*	Down	Not Down
Down	27	6
Not Down	15	30

Comments

Findings clearly show Markov transition tables are sensitive to measurement precision. Ipsative standardization of data prior to constructing a weighted transition table had no effect on the outcome of analyses.[6,7] In the present data set serial ratings were made nearly every day, however there were instances in which ratings for one or two days weren't made. Time between ratings should thus be examined as a potentially important aspect in Markov analyses.

References

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Author Notes

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