

# The Structure of *Perfect* Optimal Models with a Two-Category Class Variable and Four or Fewer Endpoints

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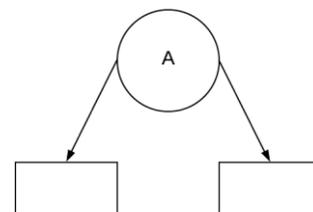
An optimal model has a specific geometric configuration defined by the number of attributes (“independent variables”—schematically illustrated using circles) and endpoints (defined by response on attribute—indicated by rectangles). Branches direct attributes to endpoints via an if/then/else-based decision rule identified by the (ODA/CTA/novometric) algorithm and operationalized vis-à-vis numerical thresholds or categorical rosters which explicitly maximize (weighted) classification accuracy. In hopes of aiding in the visualization, pursuit and discovery of perfectly accurate statistical classification models, this paper presents schematic diagrams which correspond to combinations of number of attributes and endpoints that are possible for a range of optimal models commonly reported.<sup>1</sup>

## Two-Strata Model

The least-granular model possible is a solution involving two endpoints, known as a two-strata model (Figure 1). Dummy-labeled “A” for clarity, the attribute may be ordered or (multi)categorical. Endpoints are defined by category (one for category X, one for category Y), or as being above vs. below a numerical threshold. It is not unusual to obtain perfect ODA models, that are stable in leave-one-out (LOO) jackknife analysis, in (interrupted) time-series applications.<sup>1-4</sup> Perfect two-strata models may occur in univariable analyses involving a single attribute<sup>5</sup>, or in multivariable designs with multiple attributes.<sup>6</sup> When using CTA to create propensity scores, a special-purpose procedure is needed for end-

points with perfectly classified strata.<sup>7</sup> Also, it is possible that the exact discrete 95% confidence interval for accuracy yielded by a binary CTA model for one or both endpoints overlaps perfect classification.<sup>8</sup> However, binary attributes may be insufficiently sensitive (inadequately granular) to identify true maxima.<sup>9</sup>

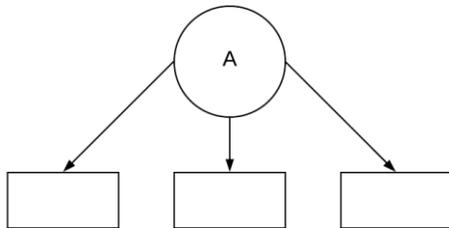
Figure 1: One-Attribute Configuration Possible for Two-Strata Optimal Classification Model



### Three-Strata Models

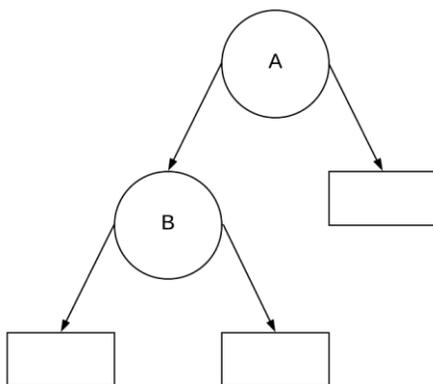
Linear and nonlinear findings of optimal models in applications with a binary class variable, a multicategorical or an ordered attribute, and a three-strata solution, are not uncommon.<sup>10-14</sup>

Figure 2: One-Attribute Configuration Possible for Three-Strata Optimal Classification Model



Among published optimal models for applications with a binary class variable and multiple categorical and/or ordered attributes, two-attribute three-strata CTA models (Figure 3) often reflect the best combination of *predictive accuracy* (assessed as ESS or PAC)<sup>1</sup> and *parsimony* (inhibiting model overfitting).<sup>15,16</sup> Attribute B may enter the model from either the left or right branch emanating from A.

Figure 3: Two-Attribute Configuration Possible for Three-Strata Optimal Classification Model



### Four-Strata Models

Increasingly complex models involving more sample strata are sometimes optimal for the

sample, and often are fertile hunting grounds for statistically-motivated (branch) sub-models that are productive in precision forecasting as well as model performance-boosting.<sup>17</sup> Four-strata models may involve one (Figure 4), two (Figures 5 and 6) or three attributes (Figure 7).

Figure 4: One-Attribute Configuration Possible for Four-Strata Optimal Classification Model

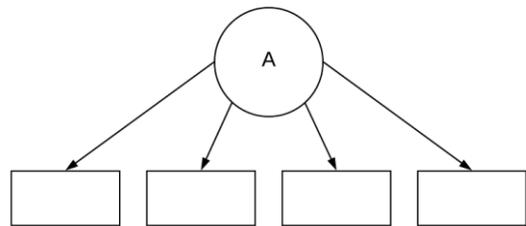


Figure 5: Two-Attribute Configuration Possible for Four-Strata Optimal Classification Model

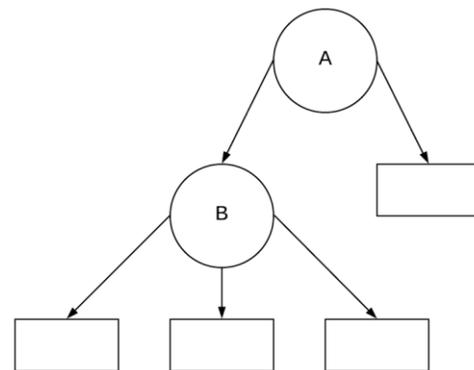


Figure 6: Two-Level Three-Attribute Configuration Possible for Four-Strata Optimal Classification Model

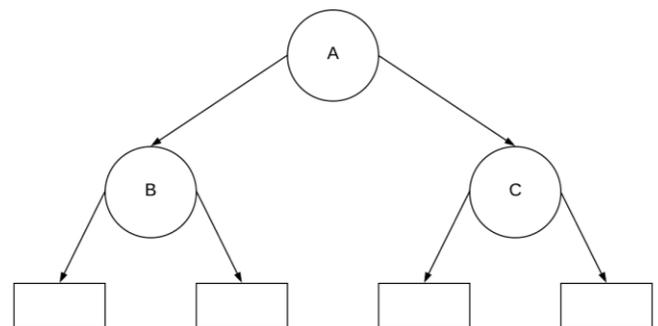
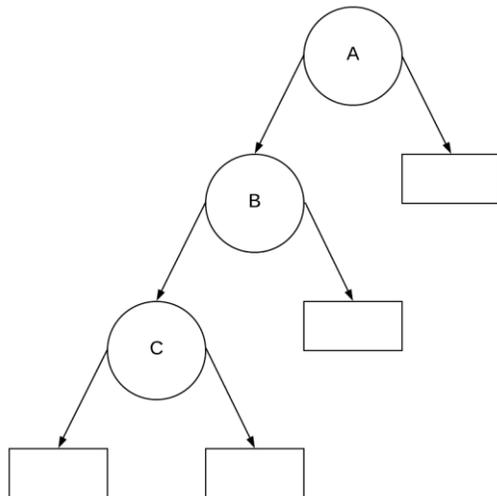


Figure 7: Three-Level Three-Attribute Configuration Possible for Four-Strata Optimal Classification Model



In Figure 5 attribute B may enter the model from the left or right branch emanating from attribute A. In Figures 6 and 7 attribute C may enter the model from the left or the right branch emanating from attribute B.

### Comments

Configurations illustrated here reflect structure typically identified by (occasionally perfect) optimal models obtained in primarily exploratory research.<sup>17</sup> Optimal exploratory methods have also been developed, such as the optimal analogue to principal components analysis, called structural decomposition analysis, which maximizes accuracy (ESS or PAC) rather than variance.<sup>18</sup> And, the minimum denominator selection algorithm<sup>19</sup> is employed to identify all of the unique, statistically viable optimal models with differing complexity and statistical power existing in the sample. Using ipsative standardization in serial designs limits or prevents paradoxical confounding.<sup>20,21</sup> The list goes on.<sup>17</sup>

By design, development of the ODA paradigm was predicated on providing unlimited flexibility to precisely, exactly model any

exploratory or confirmatory hypothesis. Early examples abound, and include Foa's "spokes of a wheel" model of the exchange of psychological and material resources (p. 196) and "circumferential rotation of a wheel" model of temporal stability of learning style (p. 218); Loo's "rotating square" model of learning style transitions (p. 217); and Goodman's "rotating triangle" model of generalizable transitions in voting intentions.<sup>10</sup> Most recently, novometric models have been shown to outperform a host of legacy methods including general linear model, logistic regression, log-linear and Cox survival analysis, interrupted time series and Markov modelling, reliability and validity analyses, structural equation modeling, and a suite of causal inference methodologies.<sup>22-32</sup> The stage is set, the tools are available, for a host of new discoveries to be made using old as well as new data.

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#### **Author Notes**

No conflict of interest was reported.