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When to Evaluate a Nonlinear Model

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Two bivariate data sets and four regression models provide the solution.

This paper is organized in consecutive sections in which—in reality, either a nonlinear or linear *relationship* underlies two variables, X and Y.

Both of the sections present consecutive findings of nonlinear and linear *models* of the relationship between X and Y.

Table 1 presents the artificial data used in this simulation study.

Table 1: Simulated Data for Exposition

Nonlinear Relationship		*Linear* Relationship	
X	Y	X	Y
10	50	10	50
12	32	11	48
20	20	18	46
25	12	26	38
30	11	34	32
35	16	38	30
40	21	44	26
50	34	48	22

Underlying *Nonlinear* Relationship

Nonlinear Model

As seen in Figure 1, data indicate one inflection point: the highest recorded value of Y (vertical axis) occurs at the lowest recorded value of X (horizontal axis); Y decreases monotonically until reaching X=30 and then reverses, increasing monotonically until X reaches its highest recorded level. In geometric terms, Y follows a parabolic path over X.

The number of inflections in Y which occur over the domain of X indicates the terms to include in the model: no inflections indicate a *linear* model with an intercept and a coefficient for X; one inflection point indicates a *nonlinear* model with an additional coefficient for X^2; two inflections indicate a *nonlinear* model with an additional coefficient for X^3; etcetera.

Visually the nonlinear model seems to be an excellent representation of the actual data. For example, to predict Y for X=30 by the equation in Figure 1, Y-Hat=0.0696*30^2–4.3491*30+80.031=12.2, a relatively close estimate of the true value 11 (Table 1). The omnibus model is statistically significant [F(2,5)=24.53, $p<0.0026$] and accurate (R^2=0.908), and both the linear (t=-6.93, $p<0.001$) and nonlinear (t=6.57, $p<0.0012$) coefficients were significant.

Linear Model

Visually the linear regression model appears as a poor representation of the data—only two data points lie in close proximity to the model. The model is not statistically significant [F(1,6)= 0.74, $p<0.43$] and very weak (R^2=0.109)—only capable of accurately predicting the mean.[1]

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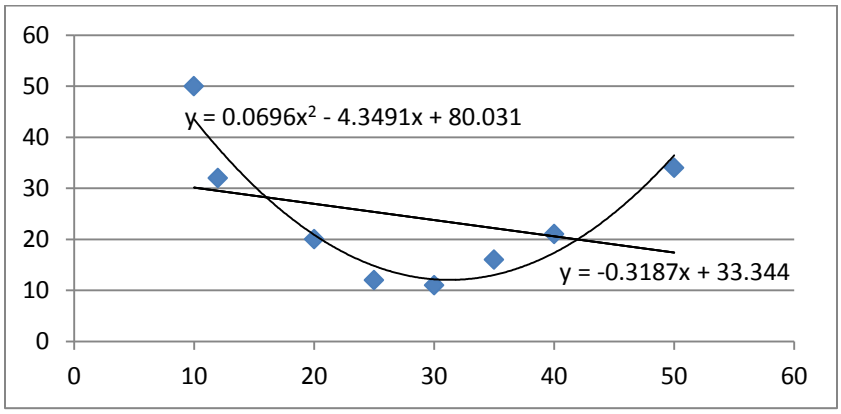
Figure 1: Nonlinear Data, Linear *vs*. Nonlinear Models



Underlying *Linear* Relationship

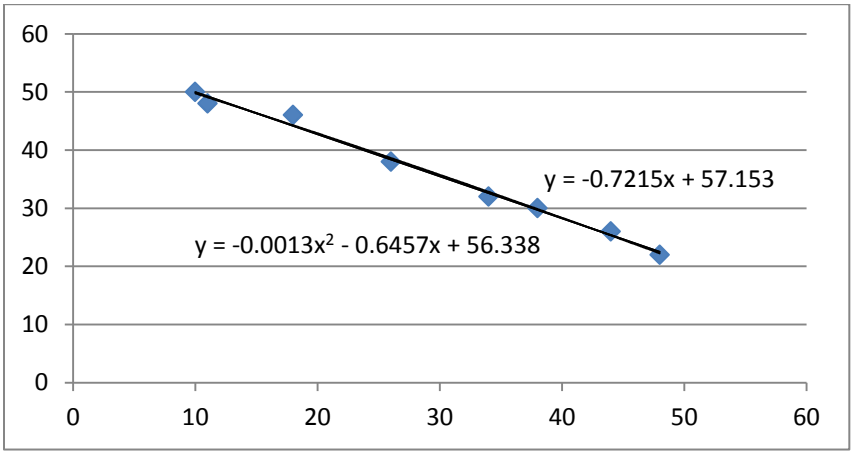
Nonlinear Model

As seen in Figure 2, data indicate what visually appears to be a nearly perfect linear effect. To predict Y for X=34 by the equation given in Figure 2, Y-Hat=-0.0013*34^2–0.6457*34+ 56.338=32.9, a relatively accurate estimate of the true value of 32 (Table 1). The omnibus model is significant [F(2,5)=337.4, p<0.0001] and accurate (R^2=0.993): although the linear coefficient was significant (t=-4.06, p<0.0098), the nonlinear coefficient was not statistically significant (p<0.65).

Linear Model

The model is statistically significant [F(1,6)= 773.33, p<0.0001] and accurate (R^2=0.992). Predicting Y for X=34 by the equation given in Figure 2, Y-Hat=-0.7215*34+57.153=32.6, a relatively accurate estimate of the true value of 32 (Table 1).

Figure 2: Linear Data, Linear *vs*. Nonlinear Models



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Comments

Results showed that if a nonlinear effect exists then a *nonlinear* model *will* find the nonlinear effect, but a *linear* model *won't* find the nonlinear effect. And, if a linear effect exists then both *linear* and *nonlinear* models *will* find the linear effect (the nonlinear model may or may not also find a nonlinear effect). This suggests it is never a bad idea to examine possible nonlinearity in one's data. Other than finding a perfect linear model, it is difficult to imagine a good reason to not check for nonlinearity in data.

Simulated data used here reflected close to perfect effects, not uncommonly obtained in engineering-related applications, but less often realized in most empirical science. Systematic manipulation of small synthetic data sets such as those used here will familiarize researchers new to nonlinear modeling with the effect of changes in data configurations upon changes in *p*-values, model coefficients, R^2, etcetera, for such general linear models. Conceptually-related research is needed to study the nonlinear performance of all linear statistical methodologies.[2-5] The finding of statistically significant linear *and* nonlinear terms for the nonlinear model when it is applied to the nonlinear data suggests a possible synergy with Markov analysis research investigating the number of processes underlying a transition table, as well as heterogeneous populations.[4-12]

It is important to recall that predictive accuracy achieved by suboptimal linear models is explicitly *optimized* using ODA to maximize either ESS or PAC (i.e., Percentage Accurate Classification).[13-27] Arguably more important, recall that explicitly *optimal* linear MultiODA models surpass the accuracy achieved by using optimized suboptimal models, and can identify models in applications for which suboptimal linear methods can find nothing.[28-33]

Perhaps it is most important to consider how many of the phenomena studied in science or events experienced in life have simple linear underpinnings? Most currently, development of a family of nonlinear optimal classification tree analysis algorithms, including novometric theory, explicitly identify the most accurate and parsimonious (non)linear models possible for a sample of data.[34-57]

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Author Notes

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