

Using ODA to Confirm a First Order Markov Steady State Process

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Sufficiently iterated over time periods a first order Markovian change process defined by a constant transition matrix yields a steady state.¹ Consecutive transition matrices are compared by Goodman’s chi-square test to assess if a steady state has been achieved.² This note demonstrates the analogous use of ODA to assess if such transition matrices differ.

Data for this exposition (Table 1) were drawn from a study tracking how British respondents

identified themselves as middle- or working-class between consecutive years.³

Table 1: Turnover Data¹ Used to Test Transition Matrix Dissimilarity

| | | Response at $t+1$ | | Chi-Square (df=1) |
|-----------------------|--------------|-------------------|---------|-------------------|
| | | Middle | Working | |
| Middle Class (t) | $t=1$ (1963) | 198 | 64 | 2.01 |
| | $t=2$ (1964) | 205 | 87 | |
| Working Class (t) | $t=1$ (1963) | 94 | 639 | 0.57 |
| | $t=2$ (1964) | 81 | 622 | |

Described by Markus¹, “The usual chi-square statistic is calculated for each section (of Table 1), and the two chi-square values are summed. Each 2x2 section has one degree of freedom, so for the sum of the two chi-squares, $df=2$Consulting a tabulation of chi-square values, one finds that neither value approaches significance ($p>0.10$), nor does their sum (thus the hypothesis of a constant transition matrix appears to be reasonable” (p. 11).

In this method chi-square is used to assess if corresponding rows of contiguous transition matrices differ—if corresponding rows can’t be discriminated, this is interpreted as confirmation that a steady state is reached.

ODA offers an exact, nonparametric analogue to Goodman’s test for this application.⁴⁻⁶ Two ODA analyses are used, consistent with the chi-square method. Table 2 summarizes ODA findings for the present data.

Table 2: Findings of ODA Tests of Transition Matrix Dissimilarity

| | | Response at $t+1$ | | ODA Results | | |
|---------------|--------------|-------------------|---------|-------------|------|-------|
| | | Middle | Working | Sensitivity | ESS | $p <$ |
| Middle | $t=1$ (1963) | 198 | 64 | 75.6 | 5.37 | 0.089 |
| Class (t) | $t=2$ (1964) | 205 | 87 | 29.8 | | |
| Working | $t=1$ (1963) | 94 | 639 | 12.8 | 1.30 | 0.471 |
| Class (t) | $t=2$ (1964) | 81 | 622 | 88.5 | | |

Note: For both ODA analyses (one for Middle Class, the other for Working Class), year was treated as a two-category class variable, and response at $t+1$ as a categorical attribute. The solution in both analyses yielding maximum ESS was: if Year=1963 predict response=Middle Class, and if Year=1964 predict response=Working Class. Using ODA individual test results are considered individually—ESS isn't summed.

Extremely weak ESS values obtained by ODA models indicate extremely weak discrimination which is possible between corresponding rows of contiguous transition matrices. For the Middle Class the ODA model is unable to correctly predict $t=2$ data, and for the Working Class the ODA model is unable to predict $t=1$ data (50% accuracy is expected by chance for both $t=1$ and $t=2$ data).

References

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Author Notes

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