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ANOVA with Three Between-Groups Factors *vs*. Novometric Analysis

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ANOVA with three between-groups factors is used to compare an ordered attribute (score) between four independent categories of class variable "A", three independent categories of class variable "B", and two independent categories of class variable "C". The novometric multiple regression analogue is demonstrated.

Described elsewhere[1], data were: "…72 scores [integers from 1 to 14] obtained from 72 different subjects, organized into twenty-four groups of three" (p. 94). Prior analysis of these data using 4 (Factor A) x 3 (Factor B) x 2 (Factor C) between-groups factorial ANOVA revealed statistically significant main effects of Factors A [$F(3,48)=3.65$, $p<0.0188$], B [$F(2,48)= 7.18$, $p< 0.0019$], and C [$F(1,48)=24.81$, $p<0.0001$], but no statistically significant interactions between Factors A, B, and/or C (p. 108). Discussion did not consider follow-up analyses needed to disentangle main effects, effect strength ("ecological significance"), or potential cross-generalizability of the findings.[2]

ANOVA designs involving two or more between-subjects factors, and one-way designs[3], are analyzed by the novometric multiple regression analogue approach. Treated as an ordered class variable, score is discriminated by globally optimal (GO) CTA treating Factors A, B and C, as well as their two- and three-way interactions as multicategorical attributes. The goal of this approach is to accurately predict observations' scores. To maximize cross-generalizability, all models were constrained to have identical ESS in training and LOO analysis.[4-10]

Presently, statistically significant models emerged for eight scores (Table 1). All eight of the models are binary parses which define two strata, thus all are comparably parsimonious.

The last model in Table 1, with a score threshold value of 12, has greatest ESS (a strong effect) and is thus the GO model in this application.[4] In this study scores >12 reflect the upper 5.6% of the sample distribution. The GO model has positive predictive value=22.2% when predicting that an individual's score will be greater than 12 (Table 2).

The first model in Table 1, with a score threshold value of 3, has second-greatest ESS (a relatively strong effect[10]): scores $\leq$3 reflect the lower 5.6% of the sample distribution. The model has negative predictive value=16.7% when predicting that an individual's score will be three or lower.

Models in Table 1 are binary parses, but initial CTA models for scores of 7 and 8 used two attributes.[11] For example, Figure 1 is the first CTA model predicting score$\leq$8. The model correctly classified 37/40 (92.5%) of scores $\leq$8, and 21/32 (65.6%) of scores >8: ESS=58.1% (a relatively strong effect). For this model, D=2.16 (D=0 when ESS=100).[12-14]

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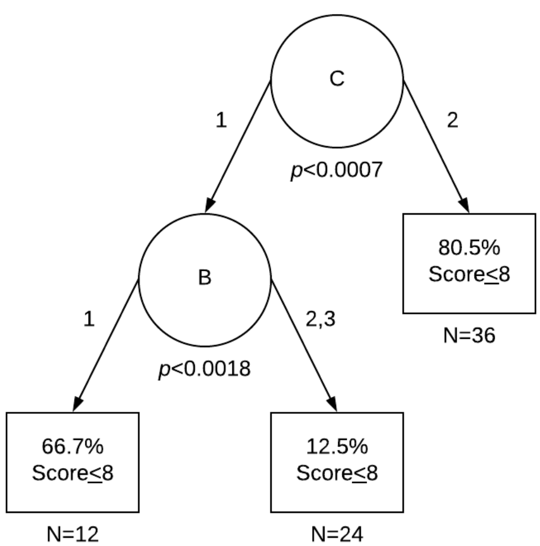
Table 1: Findings of CTA Analyses Predicting Score

		p<	
Model	ESS	Training	LOO
Predict Score≤3 if Factor B=1	70.6	0.0311	0.0104
Predict Score≤6 if Factor C=1	34.6	0.0167	0.0084
Predict Score≤7 if Factor C=1	43.3	0.0007	0.0004
Predict Score≤8 if Factor C=1	50.6	0.0001	0.0001
Predict Score≤9 if Factor C=1	39.3	0.0045	0.0022
Predict Score≤10 if Factor C=1	54.7	0.0003	0.0002
Predict Score≤11 if Factor C=1	55.4	0.0113	0.0057
Predict Score≤12 if Factor A=2, 3, or 4	79.4	0.0117	0.0030

Table 2: Confusion Matrix for GO Model (Score=12)

		Predicted Score		
		≤12	>12	Sensitivity
Actual	≤12	54	14	79.4
Score	>12	0	4	100.0
Predictive Value		100.0	22.2	

Figure 1: Initial CTA Model for Score=8 (Unrestricted Minimum Strata N)



The unrestricted minimum strata N model in Figure 1 illustrates a main effect of score C=2, and an interaction between score C=1 and scores of 1 *vs*. of 2 and 3 on Factor B.

Table 3 is the confusion matrix for this model. Note that the model has excellent predictive value—correct in more than 3 of 4 instances in which an observation is predicted by the CTA model to have a score≤8, or to have a score>8.

Table 3: Confusion Matrix for Score=8, Unrestricted Minimum Strata N Model

		Predicted Score		
		≤8	>8	Sensitivity
Actual	≤8	37	3	92.5
Score	>8	11	21	65.6
Predictive Value		77.1	87.5	

The left-most endpoint has the smallest N (12) of all strata. The optimal model having a minimum endpoint N≥13 is found vis-à-vis the minimum-denominator selection algorithm or MDSA.[15,16] This algorithm identifies the "descendant family" of optimal models in a sample. The models vary on complexity, operationalized in terms of the number of endpoints.[4,5,12]

Figure 2 is the second CTA model in the descendant family for class score=8.

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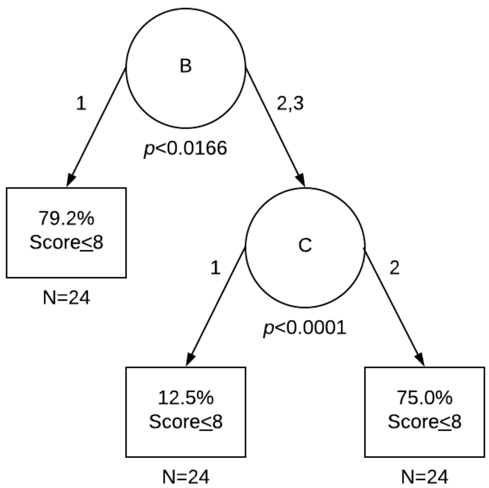
Figure 2: Second CTA Model for Score=8 (Minimum Strata N≥13)



The restricted minimum strata N≥13 model in Figure 2 illustrates a main effect of score B=1, and an interaction between score B=2 or 3 and scores of 1 *vs*. of 2 on Factor C.

The confusion matrices for the models in Figures 1 and 2 are identical, as are associated ESS and D statistics (Table 3).

The final model in the descendant family is the restricted minimum strata N≥25 model presented in Figure 3, and the confusion matrix summarizing the performance of this model is given in Table 4: for this performance, D=1.95.

Figure 3: Final CTA Model for Score=8 (Minimum Strata N≥25)

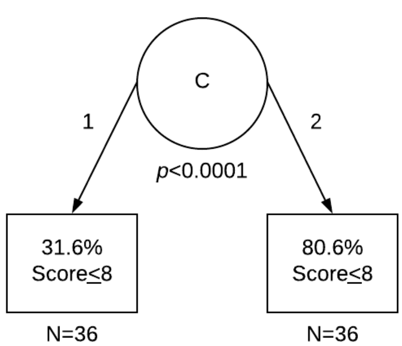


Table 4: Confusion Matrix for Score=8, Final (Minimum Strata N≥25) Model

		Predicted Score		
		≤8	>8	Sensitivity
Actual	≤8	29	11	72.5
Score	>8	7	25	78.1
Predictive Value		80.6	69.4	

Because the D statistic for the single-attribute model (1.95) is closer to 0 than the D statistic for the two-attribute models (2.16), the single-attribute model illustrated in Figure 3 is identified as being the GO model in the score=8 descendant family.[4,5,12]

Consistent with both of the two-attribute models, the one-attribute model has excellent predictive value—correct in 70% to 80% of the instances in which an observation is predicted to have a score>8, or a score≤8, respectively. The sensitivities of the one-attribute model (72.5% of scores<8, and 78.1% of scores>8, were correctly classified) are more balanced than occurred for the two-attribute models (92.5% and 65.6%, respectively).

Strategically (theoretically), concerning findings for score=8, the one-attribute model is closest to a theoretically ideal model vs. the two-attribute models. Tactically (in practice), the two-attribute models provide the greatest sensitivity for predicting score≤8, and greatest predictive value when making predictions of score>8. Conversely, the one-attribute model provides the greatest sensitivity for predicting score>8, and greatest predictive value when making predictions of score≤8. This approach offers researchers solutions for a variety of both theoretical and applied inquires.

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Author Notes

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