

# UniODA vs. *Not* Log-Linear Model: The Relationship of Mental Health Status and Socioeconomic Status

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Data are from a classic 4 x 6 contingency table used to determine the relationship (if any) between mental health status measured using four ordered categories, and socioeconomic status (SES) measured using six ordered categories.<sup>1</sup> Although ordinal data are inappropriate for analysis via chi-square-based methods, log-linear analysis was used to investigate association between mental health and SES.<sup>2-9</sup> A variety of legacy measures of association indicated “moderate” association, but differed in terms of their use of standardization, statistical reliability testing, and identification of the underlying direction (positive, negative, or non-linear) and strength of the relationship. UniODA overcomes these legacy shortcomings, as illustrated in three “standard” analysis modes: *exploratory* (a non-linear model was identified having a very weak effect, ESS = 6.4), or *confirmatory* hypothesizing either a positive (ESS = 5.6) or a negative (ESS = -3.0) relationship.

The data<sup>1</sup> (frequencies) are presented in Table 1.

Table 1: Mental Health Status and SES

SES	Mental Health Status			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
<u>1</u>	64	94	58	46
<u>2</u>	57	94	54	40
<u>3</u>	57	105	65	60
<u>4</u>	72	141	77	94
<u>5</u>	36	97	54	78
<u>6</u>	21	71	54	71

Using the log-linear approach the independence (no association) model didn’t yield a satisfactory representation of the data, indicating that a relationship exists between mental health status and SES.<sup>1</sup> Multiple legacy measures of association (Cramer’s *V*, Lambda, Goodman-Kruskal tau, Uncertainty, Kendall’s tau-*B* and tau-*C*, Gamma, Somer’s *D*, Pearson’s *r*, Spearman’s *r*, Eta) were computed in an effort to determine “how strong is the relationship, how can it be compactly described, and what is the sign of the relationship, taking into account the ordering of categories”. Clogg and Shihadeh<sup>1</sup> summarized the findings: “With the exception of the lambda measure, all indexes indicate a

relationship. The magnitude of the relationship varies from a low of about 0.008 to a high of about 0.156. Some of the measures are naturally signed and indicate a positive (or negative) relationship, others do not.” (p. 20). Additional dimensions of divergence noted between legacy measures of association were the use of statistical significance testing, explicit identification of the form of the association, and assessing the effect strength—for example, the clinical or “ecological” importance—of the finding.<sup>8,9</sup>

As illustrated below no such guess-work exists in the ODA statistical paradigm. Rather, using maximum-accuracy methods a precisely appropriate analysis exists for every specific combination of hypothesis, data geometry, and sample.<sup>8-12</sup> For every analysis the exact Type I error is computed or estimated via simulation as a permutation probability, the model explicitly identifies structure underlying the relationship, and the measure of effect strength is chance-corrected (ESS = 0 is the predictive accuracy expected by chance for the application) as well as maximum-corrected (ESS = 100 is perfect, errorless prediction).<sup>8,9</sup>

### Exploratory Analysis

The first model explicitly maximized the ESS that was obtained in exploratory (“two-tailed”) training analysis by using the UniODA<sup>8</sup> and MegaODA<sup>13-15</sup> command syntax given below (in bivariate analysis the attribute having fewest ordinal categories is generally selected as the class variable<sup>8,9</sup>):

```
OUTPUT example.out;
OPEN example.dat;
VARS mental ses;
CLASS mental;
ATTR ses;
MC ITER 25000;
LOO;
TITLE exploratory analysis;
GO;
```

The resulting non-linear exploratory ODA model was: if SES = 1 or 2 then predict mental health = 1; if SES = 4 predict mental health = 2; if SES = 3 predict mental health = 3; and if SES > 5 predict mental health = 4. In this model the people with the lowest mental health had the lowest SES, and the people with the highest mental health had the highest SES, but there was local regression<sup>8,9,16,17</sup> between the two intermediate classes (2 and 3) of mental health and SES. Application of this model to the total (training sample) yielded the confusion table in Table 2.

Table 2: Confusion Table for Training and LOO Performance of Exploratory UniODA Model

		Predicted Mental Health				Sensitivity
		<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	
<u>Actual</u>	<u>1</u>	121	72	57	57	39.4
	<u>2</u>	188	141	105	168	23.4
	<u>3</u>	112	77	65	108	18.0
	<u>4</u>	86	94	60	149	38.3

As seen, the model sensitivity for actual mental health status categories only surpassed the accuracy expected by chance<sup>8,9</sup> (25.0%) for categories 1 (39.4%) and 4 (38.3%). Integrated across all four class categories, ESS = 6.36 (a very weak effect),  $p < 0.0001$ . This indicates a statistically reliable but extremely weak effect.

### Confirmatory Analysis: Positive Association

The second model explicitly maximized ESS obtained in confirmatory (“one-tailed”) training analysis testing the *a priori* hypothesis that lower mental health is associated with lower SES, using the appended command syntax given below. The DIRectional statement indicates the attribute (SES) is hypothesized to be lower for mental health status group 1 than for mental status group 2; lower for group 2 than for group 3; and lower for group 3 than for mental status group 4.

DIR < 1 2 3 4;  
 TITLE exploratory analysis;  
 GO;

The UniODA model was: if SES = 1 then predict mental health = 1; if SES = 2 predict mental health = 2; if SES = 3 predict mental health = 3; and if SES ≥ 4 predict mental health = 4. In this linear (positive association) confirmatory model, the better one's mental health status, the better one's SES. Application of this model to the total (training sample) yielded the confusion table in Table 3.

Table 3: Confusion Table for Training and LOO Performance of Confirmatory UniODA Model Hypothesizing a Positive Association

		<u>Predicted</u> Mental Health				<u>Sensitivity</u>
		<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	
<u>Actual</u>	<u>1</u>	64	57	57	129	20.8
	<u>2</u>	94	94	105	309	15.6
	<u>3</u>	58	54	65	185	18.0
	<u>4</u>	46	40	60	243	62.5

As seen, the model sensitivity for actual mental health status categories only surpassed the accuracy expected by chance for category 4 (62.5%): ESS = 5.63 (a very weak effect),  $p < 0.0001$ . This indicates a statistically reliable but extremely weak effect.

### Confirmatory Analysis: Negative Association

The third and final model explicitly maximized ESS obtained in confirmatory training analysis testing the *a priori* hypothesis that declining mental health is associated with higher SES, using the appended command syntax given below. The DIRectional statement indicates that SES is hypothesized to be higher for mental health status group 1 than for mental status group 2; higher for group 2 than for group 3; and higher for group 3 than for group 4.

DIR > 1 2 3 4;  
 TITLE exploratory analysis;  
 GO;

The UniODA model was: if SES = 1 then predict mental health = 4; if SES = 2 or 3 predict mental health = 3; if SES = 4 or 5 predict mental health = 3; and if SES = 6 predict mental health = 4. In this linear (negative association) confirmatory model, the better one's mental health status, the worse one's SES. The application of this model to the total (training sample) yielded the confusion table in Table 4.

Table 4: Confusion Table for Training and LOO Performance of Confirmatory UniODA Model Hypothesizing a Negative Association

		<u>Predicted</u> Mental Health				<u>Sensitivity</u>
		<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	
<u>Actual</u>	<u>1</u>	46	100	172	71	11.8
	<u>2</u>	58	119	131	54	32.9
	<u>3</u>	94	199	238	71	39.5
	<u>4</u>	64	114	108	21	6.8

As seen, the model sensitivity for actual mental health status categories only surpassed the accuracy expected by chance for categories 2 (39.5%) and 3 (32.9%): ESS = -2.98 (an effect that is marginally weaker than expected by chance),  $p = 1.0$ . This indicates the absence of an effect.

### References

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- <sup>16</sup>Yarnold PR (2014). How to assess inter-observer reliability of ratings made on ordinal scales: Evaluating and comparing the Emergency Severity Index (Version 3) and Canadian Triage Acuity Scale. *Optimal Data Analysis, 3*, 42-49.
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### Author Notes

The study analyzed de-individuated data and was exempt from Institutional Review Board review. No conflict of interest was reported.

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