

UniODA vs. Legacy Bivariate Statistical Methodologies

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Research comparing the use of optimal versus legacy methods for analysis of data representing different experimental designs is on-going. This note discusses bivariate legacy statistical tools for which the alternative use of UniODA has already been demonstrated as an always valid, exact, maximum-accuracy statistical methodology.

At first blush univariate optimal discriminant analysis, called UniODA^{1,2} is an inauspiciously compact decision-making algorithm with only one objective function—to identify the most accurate classification model for any given sample of classical data. Classification models that are developed using UniODA define the statistical contextual meaning of the words “transparent” and “parsimony”.¹⁻⁵ Created using a “training sample” a UniODA model predicts the state of the class (“dependent”) variable for individual observations, on the basis of the value of one or more attributes (“independent variables”).¹

For clarity, imagine that the binary class variable is gender, having two class categories. Indicator values⁶ used in class coding are arbitrary: imagine that for males, class = 1, and for females, class = 0.

For an ordered attribute, a UniODA model finds the *threshold* that yields greatest accuracy—normed against chance—for the sample: if score \leq *threshold*, then predict that the observation is from class 1; otherwise predict that the observation is from class 0.

For a categorical attribute, a UniODA model finds the *mapping* from attribute to class

variable that yields greatest (weighted) accuracy—normed against chance—for the sample: if score = *category list*, then predict that the observation is from class 1; otherwise predict that the observation is from class 0.

Regardless of the nature of the metric used in attribute measurement, in UniODA any problem can be weighted.¹ In an experimental context, if a weight is used conceptually to define the desired objective function, then the empirical failure to weight the problem is a crucial limitation of the experimental design.¹ For example, in a study of weight loss, if the observations in the training sample (that is used to construct the model) are *unit-weighted* (i.e., are assigned equivalent weight), then UniODA will identify the model that maximizes accuracy in predicting whether or not an individual observation lost weight over the course of the study. However, if observations are instead weighted by the number of pounds that they added or subtracted over the course of the study, then weighted UniODA will identify the model that maximizes accuracy in predicting the number of pounds an individual observation lost or gained over the course of the study. Any quantitative

weight can be maximized, and synergistic interactions of quantitative weights can be maximized. For example, if it is appropriate in a study to weight observations by the amount of force that they expended, then the product of acceleration times mass would be appropriate as a measure of force by which to weight observations, and accuracy of the model in predicting expended force would be maximized.

While research comparing optimal versus legacy methods for analyzing data from different experimental designs is on-going, the present note summarizes bivariate legacy statistical tools for which the alternative use of UniODA has already been demonstrated as a valid, exact, transparent, maximum-accuracy, alternative statistical methodology.

Initial and Early Research

When the algorithm and software system were first introduced, it was already clear that UniODA was powerful—obtaining more accurate and parsimonious models, in wide domains of substantive application and experimental design, than were obtained using legacy statistical methods and software.^{1,7-10} And, it also was already clear that the UniODA algorithm not only offers advantages, but it also overcomes well-known limitations of legacy tools.^{11,12} For example, unlike UniODA, legacy tools *don't* explicitly maximize *the accuracy of the model for the data*: instead, legacy methods maximize other objective functions (e.g., variance ratios or the value of the likelihood function), and make distributional assumptions *with which data must comply* in order for the method to produce valid results.³⁻⁵ An advantage of UniODA is that the level of classification accuracy that is achieved by an “optimal” (maximum-accuracy) model¹³ is summarized using the normed effect strength for sensitivity (ESS) statistic: 0 is the classification accuracy expected by chance for the application and sample being analyzed, and 100 is perfect, errorless classification of all observations in the sample.¹ The ESS statistic enables

direct, meaningful comparison of different models with respect to a standardized measure of additional classification above and beyond what is achieved by chance.^{1,14-16}

Initial research involved comparing UniODA versus *t*-test.^{1-5,17} This was generalized to more complex general linear model designs. For example, UniODA was found to be a superior alternative to one-way^{18,19} and factorial²⁰ analysis of variance, and to analysis of covariance designed to control (eliminate) the effect of a confounding variable.²¹ Early research also investigated the use of UniODA to improve the accuracy of linear models derived using legacy methods such as logistic regression analysis, Fisher's linear discriminant analysis, log-linear analysis, and probit analysis.^{1,22-24} UniODA-based optimization methods were expanded to include ordinary least-squares (multiple) regression.²⁵⁻²⁸ While UniODA offered substantive advantages versus legacy methods—such as exact *p*-values, and maximum classification accuracy normed against chance, UniODA also eliminated issues associated with use of legacy methods—such as evaluating crucial distributional assumptions (and handling violations) underlying the validity of legacy models, model sensitivity to outlying data, and regression toward the mean.

Other early research explored the development of optimal linear multivariable models, found to be more accurate and more parsimonious than sub-optimal legacy linear models.²⁹⁻³³ However, optimal nonlinear classification tree analysis³⁴⁻³⁷ conducted for ordered class variables³⁸ identifies significantly more accurate and parsimonious models than are obtained by optimized (multiple) regression.

Evolving State-of-the-Art

Additional examples of advantages of UniODA—representing solutions to problems incurred by legacy methods—emerged for chi-square analysis³⁹⁻⁴¹, the log-linear model⁴² and logistic regression analysis.⁴³⁻⁴⁵ While UniODA

offers substantive advantages versus legacy methods—such as exact, confirmatory as well as exploratory p -values, and yielding maximum (weighted) classification accuracy normed for chance, UniODA also eliminates sticky issues associated with use of legacy methods—such as evaluating and dealing with violations of crucial distributional assumptions underlying validity, model sensitivity to skewed and/or tied data, use of dummy-variables to decompose multicategorical attributes and associated design matrix hyper-growth, and the inherent susceptibility of all linear models to paradoxical confounding.

UniODA has also been demonstrated as a superior test of null hypotheses conventionally evaluated using an expanding variety of legacy statistical tools, such as polychoric correlation⁴⁶, weighted⁴⁷ or unweighted⁴⁸ kappa, ROC analysis⁴⁹, the Bray-Curtis dissimilarity test⁵⁰, Bowker's test for symmetry⁵¹, Kendall's coefficient of concordance⁵², McNemar's test for correlated proportions^{53,54}, Mann-Whitney U test,^{55,56} and both inter-method and inter-rater reliability.⁵⁷⁻⁵⁹ Advantages of UniODA versus legacy methods include exact exploratory and confirmatory p -values, and transparent models expressed in original measurement units, that explicitly achieve maximum (weighted) classification accuracy normed against chance. UniODA also eliminates troublesome issues associated with use of legacy methods—such as evaluating and dealing with violations of crucial distributional assumptions underlying validity, model sensitivity to outlying, skewed and/or tied data, use of dummy-variables to decompose multicategorical attributes and associated design matrix hyper-growth, regression toward the mean, and the inherent susceptibility of all linear models to paradoxical confounding.

Finally, conceptually related to early work considering optimal analysis of temporal phenomena as represented in turnover tables and in Markov models of state transitions¹, recent research has extended temporal analysis to the

study of all ordered series, including single-case longitudinal series.⁶⁰⁻⁶⁵

This appears to be only the beginning of the story regarding the ability of the unassuming UniODA algorithm to supplant legacy statistical methods—in situations in which the objective function being maximized is the accuracy and parsimony of empirical models.

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Author Notes

This study involved secondary data analysis of published de-identified data and was exempt from Institutional Review Board review.

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